FINAL EXAMINATION

December 8, 2010, 2 – 3:30 pm

Prof. G.C. Temes

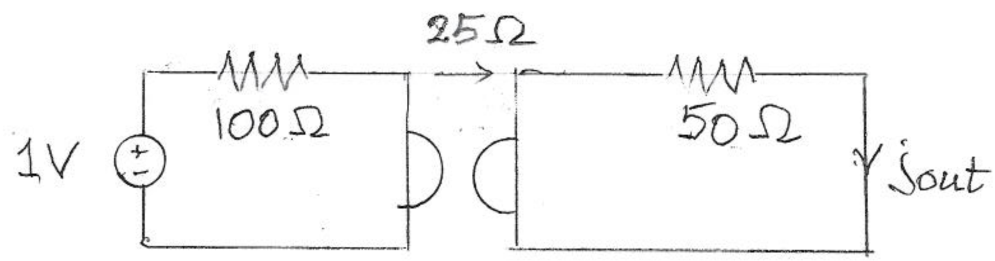
Open book

paso

1. A Butterworth filter has been designed for a passband ripple of 0.15 dB and a minimum stopband attenuation of 45 dB. Now it needs to be redesigned for a new ~~stop~~band ripple of 0.10 dB, *with the same order and selectivity.* What will be the new value of the minimum stopband loss?

2. a. Find the adjoint network of a gyrator with resistance *R.* Express the sensitivity to a change in *R* in terms of the gyrator currents in the physical and adjoint networks.

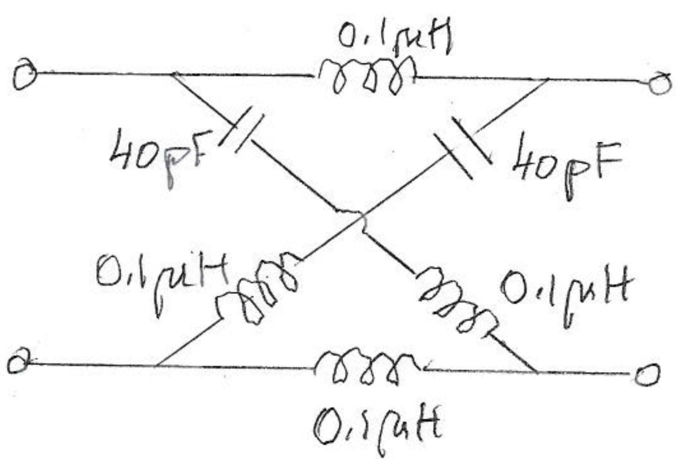
b. Find the sensitivities of the output current j_{out} to variations of the terminations and the gyration resistance *R* in the circuit shown.

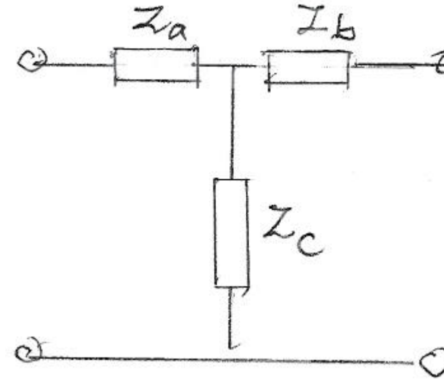


3. The two-ports shown below are equivalent. Find

a. the impedances Z_a, Z_b, *and* Z_c ;

b. the scattering matrix at ω_1 = 500 Mra/s, when the circuit is terminated by two 50 ohm resistors.





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pass

1. A Butterworth filter has been designed for a passband ripple of 0.15 dB and a minimum stopband attenuation of 45 dB. Now it needs to be redesigned for a new ~~stop~~band ripple of 0.10 dB, *with the same order and selectivity.* What will be the new value of the minimum stopband loss?

1. k_1^2 must remain unchanged, so

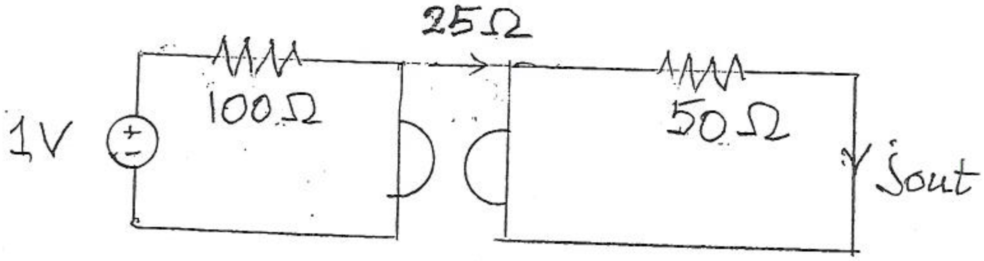
$$k_1^2 = \frac{10^{\alpha_{p1}/10}-1}{10^{\alpha_{s1}/10}-1} = \frac{10^{0.015}-1}{10^{4.5}-1} = \frac{10^{0.01}-1}{10^{\alpha_{s2}/10}-1}$$

$$10^{\alpha_{s2}/10} = \cancel{1} + \frac{10^{0.01}-1}{10^{0.015}-1}\left(10^{4.5}-\cancel{1}\right)$$

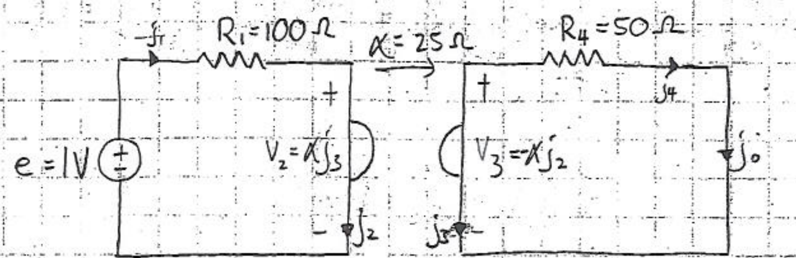
$$\alpha_{s2} \cong 45 - 1.77 = 43.23 \text{ dB}$$

2. a. Find the adjoint network of a gyrator with resistance R. Express the sensitivity to a change in R in terms of the gyrator currents in the physical and adjoint networks.

b. Find the sensitivities of the output current j_{out} to variations of the terminations and the gyration resistance R in the circuit shown.

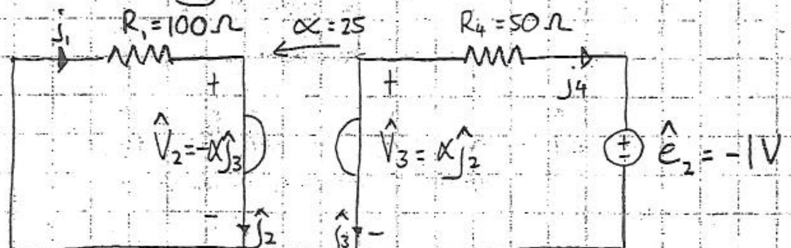


2.



R_1, R_4 & α ± 10%

And the corresponding adjoint network



From the circuit,

$$j_1 = j_2 = \frac{e - V_2}{R_1} = \frac{1 - 25 j_3}{100} = \frac{1}{100} - \frac{j_3}{4} \quad (1)$$

$$j_3 = -j_4 = \frac{-V_3}{R_4} = \frac{-25 j_2}{50} = \frac{-j_2}{2} \quad (2)$$

$$j_3 = \frac{j_2}{2} = \frac{1}{2}\left(\frac{1}{100} - \frac{j_3}{4}\right)$$

$$2j_3 = \frac{1}{100} - \frac{j_3}{4}$$

$$\underline{\underline{j_3 = \frac{1}{225}\ A}}$$

$$\Rightarrow \underline{\underline{j_4 = -j_3 = -\frac{1}{225}\ A}}$$

$$\underline{\underline{j_2 = 2j_3 = \frac{2}{225}\ A = j_1}}$$

From the adjoint network,

$$\hat{j}_1 = \hat{j}_2 = \frac{-\hat{V}_2}{R_1} = \frac{\alpha \hat{j}_3}{R_1} = \frac{25\hat{j}_3}{100} = \frac{1}{4}\hat{j}_3$$

$$\& \quad \hat{j}_3 = -\hat{j}_4 = \frac{\hat{e}_2 - \hat{V}_3}{R_4} = \frac{\hat{e}_2 - \alpha\hat{j}_2}{R_4} = \frac{-1 - 25\hat{j}_2}{50} = -\frac{1}{50} - \frac{1}{2}\hat{j}_2$$

$$\Rightarrow \quad j_2 = \frac{1}{4}\left(-\frac{1}{50} - \frac{1}{2}\hat{j}_2\right)$$

$$4.5 j_2 = -\frac{1}{50}$$

$$\underline{\underline{\hat{j}_2 = -\frac{1}{225}\ A = \hat{j}_1}}$$

$$\underline{\underline{\hat{j}_3 = 4 j_2 = -\frac{4}{225}\ A}} = \hat{j}_4$$

To find the sensitivities,

$$\frac{\partial j_0}{\partial R_1} = -\hat{j}_1 j_1 = -\left(-\frac{1}{225}\right)\left(\frac{2}{225}\right) = \frac{2}{50625}\ A/\Omega$$

$$\frac{\partial j_0}{\partial R_4} = -\hat{j}_4 j_4 = -\left(-\frac{4}{225}\right)\left(-\frac{1}{225}\right) = \frac{4}{50625}\ A/\Omega$$

$$\frac{\partial j_o}{\partial \alpha} = + \hat{j}_3 j_2 - \hat{j}_2 j_3 = +\left(-\frac{4}{225}\right)\left(\frac{2}{225}\right) - \left(-\frac{1}{225}\right)\left(\frac{1}{225}\right)$$

change in direction of α

$$= \frac{7}{50625}\ A/\Omega$$

Also,

$$\Delta R_{1max} = 100 \times 0.1 = 10\,\Omega$$

$$\Delta R_{4max} = 50 \times 0.1 = 5\,\Omega$$

$$\Delta \alpha_{max} = 25 \times 0.1 = 2.5\,\Omega$$

$$\Rightarrow |\Delta j_o|_{max} = \left|\frac{\partial j_o}{\partial R_1}\Delta R_{1max}\right| + \left|\frac{\partial j_o}{\partial R_4}\Delta R_{4max}\right| + \left|\frac{\partial j_o}{\partial \alpha}\Delta \alpha_{max}\right|$$

$$= \left(\frac{2}{50625} \cdot 10\right) + \left(\frac{4}{50625} \cdot 5\right) + \left(\frac{7}{50625} \cdot 2.5\right)$$

$$= 1.1358 \times 10^{-3} A$$

$$\approx 1.136\ mA$$

$\therefore$ The range of j_o is $\pm 1.1358\,mA$

$$j_o = -\frac{1}{225} A = -4.4\dot{4}\ mA$$

$$j_o = -4.44\dot{4} \pm 1.136\ (mA)$$

$\Rightarrow$ The range of j_o is

$$+5.58\ mA \leq j_o \leq +3.31\ mA$$

3. The two-ports shown below are equivalent. Find

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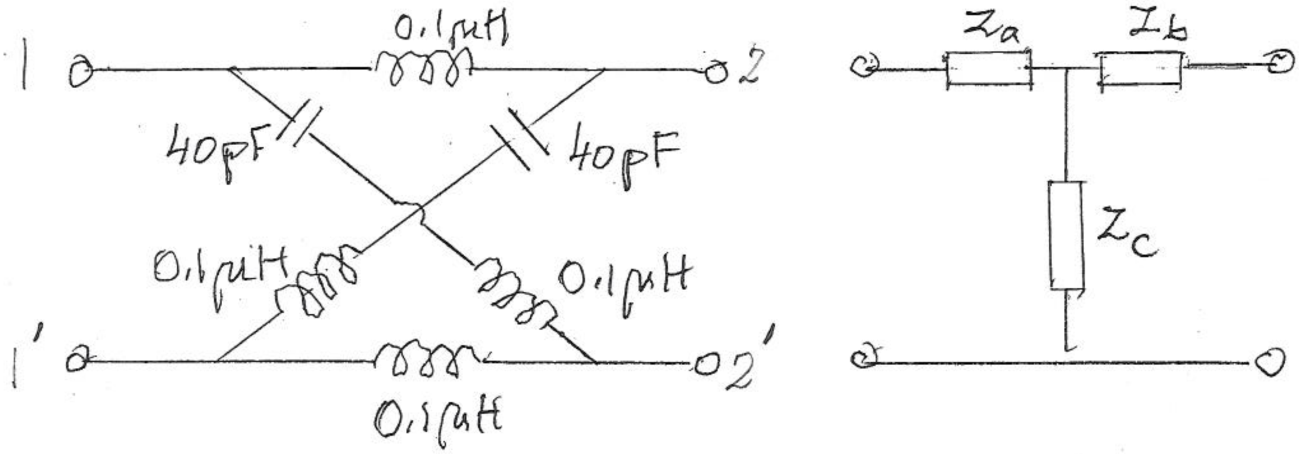
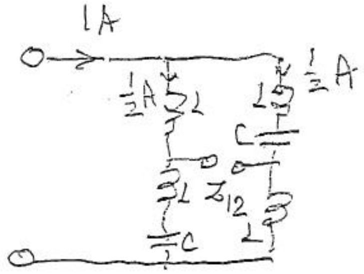
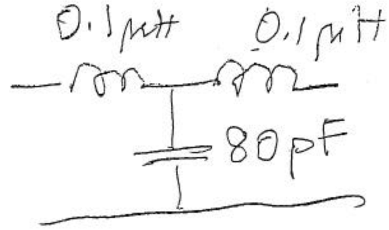


3.



$\Rightarrow$



$$z_{12} = \frac{1}{2}\left(sL + 1/sC - sL\right) = 1/s2C = z_c$$

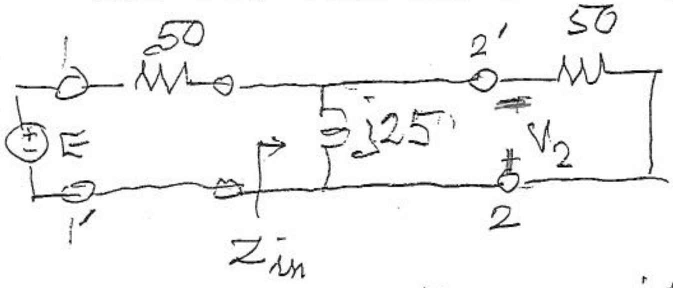
$$z_{11} = \frac{1}{2}\left(s2L + 1/sC\right) = sL + \frac{1}{s2C} = z_a + z_c$$

$$z_a = sL = z_b$$

~~At~~ $500\ \text{Mra/s} = \omega_1$, $j\omega_1 L = j50\,\Omega$

$$\frac{1}{j\omega_1 C} = -j/(5\times10^{8} \times 4\times10^{-11}) = -j50\,\Omega$$

So the lattice becomes



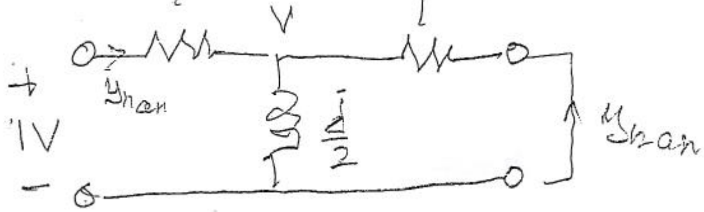
$$z_{in} = \frac{j25\times 50}{j25 + 50} = \frac{j50}{2+j}$$

$$S_{11} = S_{22} = \frac{j50 - 50(2+j)}{j50 + 50(2+j)} = \frac{-100}{100+100j} = \frac{-1}{1+j} = \frac{-1+j}{2}$$

$$S_{21} = S_{12} = \frac{-2z_{in}}{50 + z_{in}} = \frac{-j100}{50(2+j) + j50} = \frac{-j}{1+j}$$

$$= \frac{-1-j}{2}$$

Or



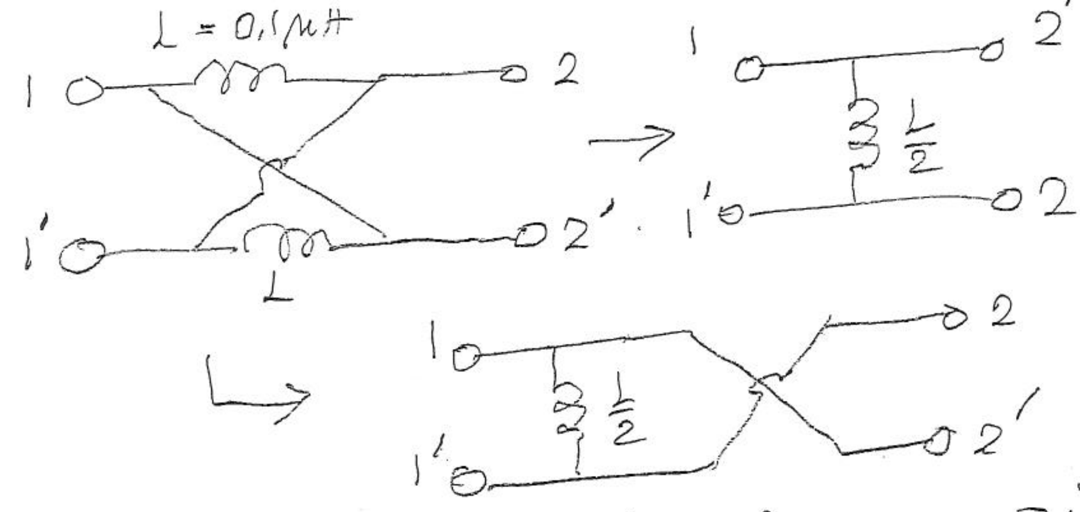
$$y_{11an} = y_{22an} = \frac{1}{1+\frac{j/2}{1+j/2}} = \frac{1+j/2}{1+j}$$

$$y_{12an} = y_{21,an} = +V = +y_{11an}\frac{j/2}{1+j/2} = +\frac{j/2}{1+j}$$

$$S_{11} = S_{22} = 1 - 2y_{11an} = 1 - \frac{2+j}{1+j} = \frac{-1}{1+j} = \frac{-1+j}{2}$$

$$S_{12} = S_{21} = -2y_{12an} = \frac{-j}{1+j} = -\frac{1+j}{2}$$

Note: lattice becomes



Causing S_{12} & S_{21} to change sign!